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Comparison between a Terramechanics Model and a Continuum Soil Model Implemented within the Absolute Nodal Coordinate Formulation

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ABSTRACT

In order to determine the best approach for the modeling of tracked vehicle-terrain interactions, a comparison is made between terramechanics and continuum mechanics plasticity soil models that can be used in vehicle dynamic simulations. The absolute nodal coordinate formulation (ANCF) which can be used in multibody system (MBS) dynamics to model large rotations and large deformations will provide a novel framework for this comparison. First, a brief review of the analytical derivation and implementation of a terramechanics soil model for drawbar pull-slip and pressure-sinkage are presented in order to establish the basic assumptions underlying this model. An assumed shape and mass for the vehicle-terrain interface and an empirically derived constitutive model are two clear assumptions inherent in terramechanics based approaches. The (modified) Bekker soil models are integrated finite element (FE) ANCF interpolations to determine the generalized forces. Continuum mechanics soil models that are suited for tracked vehicle-terrain interaction are identified and integrated into the internal force calculation of ANCF MBS algorithms. The paper discusses important fundamental issues that must be addressed when implementing continuum mechanics-based soil models as well as terramechanics models into MBS algorithms for modeling complex tracked vehicle/soil interactions. The improvement of the vehicle-terrain interface estimation resulting from FE methods can provide a terramechanics approach which may be scalable to differing vehicles and loadings. It is found that the assumed existence, uniqueness, and evolution of the yield surface(s) of continuum soil models play a critical role in predicting the soil behavior while also providing a rational method for improvement of soil constitutive modeling.

1 INTRODUCTION

In the study of vehicle-terrain interactions, there are two common methods and models used: terramechanics and continuum mechanics soil models. Broadly speaking, terramechanics is the study of the relationships between a vehicle and its environment. Some of the principal concerns in terramechanics are developing functional relationships between the design parameters of a vehicle and its performance with respect to its environment, establishing appropriate soil parameters, and promoting rational principles which can be used in the design, and evaluation of vehicles (Wong, 2010). The continuum mechanics approach is comparatively new in the application to the study of vehicle-terrain interaction. This method is of special advantage, since the deformations of

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the wheel, tire and ground can be captured as the result of their mutual interaction. The continuum mechanics approach is especially suitable for clearing up the phenomena concerning deformations, such as the effect of inflation pressure, or bulldozing resistance. Additionally, the continuum mechanics approach provides new possibilities to investigate the pressure transfer onto and into the ground, and therefore on soil compaction (Schmid, 1995). Terramechanics models can be categorized into empirical and analytical models.

Empirical terramechanics models use established experimental measurements of appropriate parameters, properties, and behaviors of soil; these experimental results are then used to establish empirical relationships that could be used to predict at least qualitatively the response of soils under various conditions (Bekker, 1969). Parametric models, which are based on experimental work and have been widely used, offer practical means by which an engineer can qualitatively evaluate tracked vehicle performance and design.

Research in the field of terramechanics has continued over many decades. Since the tractive performance of a vehicle depends significantly on the normal and shear stress distribution at the soil-vehicle interface, many attempts have been made to predict such distributions beneath tracks and wheels (Schmid, 1995). The following gives a brief overview of the developments in terramechanics and continuum-based soil models.

Bekker (1956, 1960) first proposed a theoretical study regarding the pressure-sinkage relationship of terrain, as well as the response to repeated loading and unloading. Janosi and Hanamoto (1961) proposed a model for predicting the ground pressure distribution beneath a rigid track. They assumed the normal stress distribution to be higher at the rear end of the track. The predicted ground pressure distribution was trapezoidal with higher pressure at the rear end of the track. Reece (1965) improved Bekker's model by making the parameters dimensionless. This single equation could then account for different plate shapes.

Garber and Wong (1981) developed analytical methods for predicting the pressure distribution beneath tracks taking into account all major design parameters of the vehicle as well as the pressure-sinkage characteristics of the soil. Wong et al. (1984) continued this work and established a model for the prediction of the distribution of ground pressure and the tractive performance of a tracked vehicle, including the effects of repetitive loading. Wong and Huang (2006) evaluated the tractive performance of wheeled and tracked vehicles assuming that the tracks on a tracked vehicle have an essentially flat and rectangular contact area with uniform normal pressure and the same contact length, as well as assuming the vehicle weight is uniformly distributed among the tracks.

Similarly, Okello (1994, 1998) suggested a traction model that takes into consideration soil recovery for the repetitive loading occurring under road wheels and rubber tracks. Rubinstein (2007) modified Bekker's original model by adding a viscous friction element which allows modeling the dependence of the normal pressure on the sinkage velocity. Recently Irani (2011) has proposed a new form for the pressure sinkage relationship based on the expression proposed by Reece (1965) to capture the dynamic oscillations observed for a wheel with grousers.

Similarly, research into continuum-based soil models has continued over many decades. At present, there are many continuum mechanics-based soil models that employ different assumptions, of which some are suited for finite element implementations. The following gives an introduction to some of the more common continuum-based single-phase plasticity models.

The Mohr-Coulomb model is one of the oldest and best-known models for an isotropic soil (Goldscheider, 1984). Initially the yield surface was used as a failure envelope, and still is in geotechnical practice. It was later adopted as a yield surface for plasticity models. In two dimensions, the yield surface of the Mohr-Coulomb model is defined by a linear relationship between shear stress and normal stress. The failure envelope defined by the Mohr-Coulomb model includes discontinuous slopes between failure surfaces.

A simpler method to handle the discontinuities is to use a smooth approximation to the yield surface. Drucker and Prager (1952) initially proposed a cone in principal stress space , by adding a pressure-dependent term to the classical von Mises yield surface. Like von Mises plasticity, one-step return-mapping can be achieved for linear hardening, making the model quite efficient to implement. While the associative model over predicts dilatation, non-associative versions correct this (Drucker et al., 1952). Initially developed as an elastic-perfectly plastic model, i.e. with no change in the yield surface on loading, researchers later added hardening of the yield surface parameters to the model in various forms. See, for example, Vermeer and de Borst (1984) for a relatively sophisticated phenomenological hardening model.

The modified Cam-Clay (MCC) model by Roscoe et al. (1963) is based on critical state theory and was meant to capture the properties of near-normally consolidated clays under triaxial compression test conditions. The yield surface is assumed to have an elliptical shape that may be expanded with the increase of volumetric strain. Cam-Clay models can predict failure and the nonlinear stress-path dependent behaviors prior to failure accurately, especially for clay type soils (DiMaggio and Sandler, 1971).

Cap plasticity models were developed to address the shortcomings of the Cam-Clay type models, especially for high-pressure behavior. Drucker et al. (1957) first proposed that "successive yield surfaces might resemble an extended Drucker-Prager cone with convex end spherical caps" (Chen and Baladi, 1985). As the soil undergoes hardening, both the cone and the end cap expand. This has been the foundation for numerous soil models.

The Sandia GeoModel builds on the Cap model with some modifications. It is capable of capturing a wide variety of linear and nonlinear model features including Mohr-Coulomb and Drucker-Prager plasticity depending on the model parameters incorporated. Unlike the Cap model, the cap surface and shear yield surface are connected in a smooth manner, and the model also accounts for differences in triaxial extension and compression strength using either a Gudehus or William-Warnke modifying function.

Plasticity models such as those described above do not include strain-rate dependent behavior often observed in soils under rapid loading. These viscous effects are more pronounced in the plastic region of most clay soils and rate-independent elastic response is generally adequate for practical engineering applications (Perzyna, 1966; Lorefice, 2008). The models described above can be modified to account for rate-dependent plastic effects. Such viscoplastic models are more accurate under fast loading conditions. However, it is difficult to determine the correct value of the material time parameter if the stress history is not known.

Soil is not always an isotropic material. Layering and fracture networks, as well as compaction and other history effects may give the soil higher strength or stiffness in certain directions. Often the effects impart different strength and stiffness in one plane. Anisotropy may also be addressed using fabric tensors (Wan and Guo, 2001). Other anisotropic models include the work of Whittle et al. (1994), and the S-CLAY 1 model (Wheeler et. al., 2003), which builds on the MCC model. Detailed review of anisotropic soil models is beyond the scope of this article, however, and the reader is referred to the above references.

This paper aims to compare an existing terramechanics soil model with a continuum mechanics-based soil model. A brief review of the analytical derivation and implementation of a Bekker's terramechanics soil model for pressure-sinkage and drawbar pull-slip is presented. Their suitability for incorporation into FE/MBS simulation algorithms is also discussed. Section 2 outlines the analytical approaches used in terramechanics. Section 3 describes the continuum mechanics-based soil models. Section 4 describes the ANCF implementation. The comparison of the two models is given in Section 5. Section 6 offers a summary and describes the direction of future work.

2 TERRAMECHANICS BASED SOIL MODELS

Three common formulations of the pressure-sinkage relationship are widely used in vehicle-terrain interaction investigations.

The first common pressure-sinkage relationship for mineral terrains was proposed by Bekker (1960)

$$p=\left(k_c/b+k_\varphi\right)z^n=k_{eq}z^n \tag{1}$$

where p is pressure, b is the radius of a circular plate or the smaller dimension of a rectangular plate, n is a non-dimensional soil constant related to the characteristics of the soil being modeled, k_c and k_ϕ are pressure-sinkage parameters related to the cohesion and the angle of shearing of the material, and z is the sinkage.

Besides what has been introduced, Equation (1) is now widely used in track-terrain interaction studies. For examples of such studies refer to Wills (1963), Ryu et al. (2003), Garber (1984), Okello (1994, 1998), Rubinstein et al. (2007), Park et al. (2008). Similarly for tire-terrain interactions see Mao (2008), Sandu et al. (2010), Schwanghart (1991), Harnisch (2005), Gibbesch and Schafer (2005), Li and Sandu (2007), Meirion-Griffith and Spenko (2011) , Li and Sandu (2007), and Ishigami et al. (2007).

Another common formulation of the pressure-sinkage relationship was proposed by Reece (1965)

$$p=\left(ck_c^{'}+\gamma bk_\phi^{'}\right)\left(\frac{z}{b}\right)^n \tag{2}$$

where, $k_c^{'}$ and $k_\phi^{'}$ are dimensionless soil parameters which replace Bekker's k_c and k_ϕ parameters. Equation (2) can be used on wheel-terrain interaction such as in Irani (2011) and tire-terrain interaction such as in Senatore (2011).

Similarly, among the more common formulations of the pressure-sinkage relationships are (Reece, 1965)

$$\sigma_1=\left(k_1+k_2b\right)\left(\frac{r}{b}\right)^n\left(\cos\theta-\cos\theta_1\right)^n \tag{3}$$

$$\sigma_2=\left(k_1+k_2b\right)\left(\frac{r}{b}\right)^n \cdot\left[\cos\left(\theta_1-\frac{\theta}{\theta_m}\left(\theta_1-\theta_m\right)\right)-\cos\theta_1\right]^n \tag{4}$$

where σ_1 is the forward part of the normal stress, σ_2 is the rear part of the normal stress, n is the sinkage exponent, k_1 and k_2 are pressure sinkage moduli, b is the wheel width, r is the wheel radius, θ_1, θ_m are the entry angle and the angular location of the maximum normal stress, respectively. Equation (3) and (4) can be used in wheel-terrain interactions such as in Wong et al. (1967), Ding et al. (2009), and Shibly et al. (2005). For a flexible tire acting on deformable terrain, Equation (3) and (4) can also be used if radius r is taken to be the effective radius (Senatore et al., 2011).

For 'plastic' soils which do not exhibit a 'hump' of maximum shear stress, the following modified version of Bekker's equation containing only one constant was proposed by Janosi and Hanamoto (1961) and is widely used in practice:

$$s / s_{\max} = 1 - e^{\ j/K} \qquad (5)$$

where K is usually referred to as the shear deformation parameter. It is a measure of the magnitude of the shear displacement required for the development of maximum shear stress.

It has been found that the Mohr-Coulomb equation describes the normal stress-shear stress displacement relation adequately in many cases

$$s_{\max} = c + p \tan \phi \qquad (6)$$

In this equation, $s_{\max}$ is the maximum shear stress, p is the normal stress, and c and ϕ are the cohesion (or adhesion) and angle of shearing resistance, respectively. In deriving the values of c and ϕ from measured shear data, the effect of grouser height and of the shear ring should be taken into consideration (Reece, 1964).

The tractive performance of a link can be predicted once the normal pressure and shear stress distribution are known at a given slip, it is usually characterized by its motion resistance, tractive effort, and drawbar pull (the difference between tractive effort and motion resistance) as functions of slip.

If it is assumed that a track link has a rectangular contact area with uniform normal pressure, the same contact length l, and the link and terrain is horizontal, then under steady-state operating conditions, the link tractive effort F and drawbar pull F_d at a given slip i can be given by (Bekker, 1969)

$$\begin{aligned} F &= b\int_0^l \left(c + p\tan\phi\right)\left(1 - e^{-ix/K}\right)dx \\ &= bl\left(c + p\tan\phi\right)\left[1 - \frac{K}{il}(1 - e^{-il/K})\right] \end{aligned} \qquad (8)$$

$$F_d = F - R \qquad (9)$$

where b is the contact width of the track link, l is the length of track link in contact with the terrain, p is the normal pressure, and R is the motion resistance of track link.

3 CONTINUUM MECHANICS-BASED SOIL MODELS

The original Cam-Clay model has not been as widely used for numerical predictions as the modified Cam-Clay (MCC). The qualifier “modified” is often dropped when referring to the modified Cam-Clay model (Wood, 1990). Cam-Clay models can predict failure and the nonlinear stress-path dependent behaviors prior to failure accurately, especially for clay type soils (DiMaggio and Sandler, 1971). The function for the yield surface of the infinitesimal MCC model is defined as

$$q^2 - M^2\left[p\left(p_c - p\right)\right] = 0 \qquad (10)$$

Here, q is the deviatoric stress, p is the effective mean stress, the pre-consolidation stress p_c acts as a hardening parameter, and the stress ratio $M = q/p$ at critical state is related to the angle of friction through the relationship $M = 6\sin(\phi)/(3+\sin(\phi))$. Cam-Clay models began with infinitesimal strain assumptions and have been developed to the case of finite strains in Borja and Tamagnini(1996). Furthermore, Cam-Clay models have been extended to capture the cyclic behavior of soils (Carter et al., 1982).

The constitutive equations that govern the behavior of the hyperelastic elastoplastic finite deformation Cam-Clay model (Borja et al., 1996) are summarized as follows. The yield function is defined by

$$f = f\left(P, Q, P_c\right) = \frac{Q^2}{M^2} + P(P - P_c) \qquad (11)$$

where P, Q, P_c, and M are the finite deformation analogs of the parameters defined for the infinitesimal case using the Kirchhoff stress tensor. The hardening law expressed in terms of the plastic component of the volumetric strain is given by

$$\frac{\dot{P}_c}{P_c} = -\Theta\dot{\varepsilon}_v^p, \quad \dot{\varepsilon}_v^p = \frac{\dot{J}^p}{J^p}, \quad \Theta = \frac{1}{\hat{\lambda} - \hat{\kappa}} \qquad (12)$$

where J^p is the plastic component of the Jacobian. The parameters $\hat{\lambda}$ and $\hat{\kappa}$ can be calculated from the corresponding infinitesimal model analogs. The discrete flow rule at time t_{n+1} for implicit time integration in the space defined by the elastic Eulerian logarithmic stretches can be written as

$$\boldsymbol{\varepsilon}_{n+1}^{e} = \boldsymbol{\varepsilon}_{n+1}^{e\,tr} - \Delta\phi_{n+1} \left.\frac{\partial g}{\partial \boldsymbol{\beta}}\right|_{n+1} \qquad (13)$$

where $\boldsymbol{\beta}$ is the Kirchhoff stress tensor, and $\Delta\phi$ is a plastic multiplier.

The above equations can be shown to lead to the following set of equations that can be used to define a scalar return mapping algorithm (Borja, 1998) in the invariants of the elastic logarithmic stretches

$$\left.\begin{aligned} &\varepsilon_v^e = \varepsilon_v^{e\,tr} - \Delta\phi \frac{\partial g}{\partial P}, \quad \varepsilon_s^e = \varepsilon_s^{e\,tr} - \Delta\phi \frac{\partial g}{\partial Q} \\ &f\left(P, Q, P_c\right) \leq 0, \Delta\phi \geq 0, \Delta\phi f\left(P, Q, P_c\right) = 0 \end{aligned}\right\} (14)$$

An example implicit integration scheme for the finite deformation Cam-Clay plasticity soil model can be developed by considering a set of simultaneous nonlinear equations. An application of the Newton-Raphson method can be used to solve this set of nonlinear equations. To this end, the residual vector **r** and the vector of unknowns **x** are written as follows:

$$\mathbf{r}^p = \begin{bmatrix} \varepsilon_v^e - \varepsilon_v^{e\,tr} + \Delta\phi \dfrac{\partial g}{\partial P} \\ \varepsilon_s^e - \varepsilon_s^{e\,tr} + \Delta\phi \dfrac{\partial g}{\partial Q} \\ f \end{bmatrix}, \quad \mathbf{x}^p = \begin{bmatrix} \varepsilon_v^e \\ \varepsilon_s^e \\ \Delta\phi \end{bmatrix} \qquad (15)$$

The Newton-Raphson solution procedure requires the iterative solution of the algebraic system $\left(\partial \mathbf{r}^p / \partial \mathbf{x}^p\right) \Delta\mathbf{x}^p = -\mathbf{r}^p$, where $\Delta\mathbf{x}^p$ is the vector of Newton differences. A closed form expression for the consistent tangent operator $\left(\partial \mathbf{r}^p / \partial \mathbf{x}^p\right)$ can be found and the algorithm can be made more efficient by the application of the static condensation technique (Borja et al., 1996).

4 ANCF IMPLEMENTATION

The FE implementation of the soil mechanics plasticity equations requires the use of an approach that allows employing general constitutive models. The vehicle/soil interaction can lead to a significant change in geometry that cannot be captured using finite elements that employ only translational displacement coordinates without significant refinement. In some soil applications, such a significant change in geometry may require the use of elements that employ gradients and accurately capture curvature changes. This requirement can be met using the FE *absolute nodal coordinate formulation* (ANCF).

ANCF finite elements do not employ infinitesimal or finite rotations as nodal coordinates; instead, absolute slopes and displacements at the nodal points are used as the element nodal coordinates. The position vector $\mathbf{r}^j$ of an arbitrary point on element j can be defined in a global coordinate system XYZ as $\mathbf{r}^j = \mathbf{S}^j\left(x^j, y^j, z^j\right)\mathbf{e}^j(t)$. In this equation, x^j, y^j, and z^j are the element spatial coordinates, $\mathbf{S}^j$ is the shape function matrix, $\mathbf{e}^j$ is the vector of element nodal coordinates, and t is time. The nodal coordinate vector $\mathbf{e}^{jk}$ at node k can be defined as follows (Shabana, 2005)

$$\mathbf{e}^{jk} = \begin{bmatrix} \mathbf{r}^{jkT} & \left(\dfrac{\partial \mathbf{r}^{jk}}{\partial x^j}\right)^T & \left(\dfrac{\partial \mathbf{r}^{jk}}{\partial y^j}\right)^T & \left(\dfrac{\partial \mathbf{r}^{jk}}{\partial z^j}\right)^T \end{bmatrix}^{\mathrm{T}} \qquad (16)$$

Fully parameterized ANCF finite elements allow using a general continuum mechanics approach to define the Green-Lagrange strain tensor $\varepsilon = \left(\mathbf{J}^{\mathbf{T}}\mathbf{J} - \mathbf{I}\right)/2$, where $\mathbf{J}$ is the matrix of position vector gradients. In dynamic soil problems, ANCF leads to a constant inertia matrix and to zero Coriolis and centrifugal forces. The mass matrix obtained using ANCF finite elements can always be written a $\mathbf{M}^{\mathbf{j}} = \int \rho^j \mathbf{S}^{j^T} \mathbf{S}^j dV^j$, where ρ^j and V^j are, respectively, the initial mass density and initial volume of the finite element. ANCF finite elements allow for straight forward implementation of general constitutive models including the continuum mechanics-based soil model discussed in this paper.

For a finite element or deformable body, the principle of virtual work can be written using the reference configuration as

$$\int_V \rho \ddot{\mathbf{r}}^T \delta\mathbf{r} dV + \int_V \boldsymbol{\sigma}_{P2} : \delta\boldsymbol{\varepsilon} dV - \int_V \mathbf{f}_b^T \delta\mathbf{r} dV = 0 \qquad (17)$$

In this equation, V is the initial or reference volume, ρ is the initial mass density, **r** is the global position vector of an arbitrary point, $\boldsymbol{\sigma}_{\mathbf{p2}}$ is the second Piola Kirchhoff stress tensor, $\boldsymbol{\varepsilon}$ is the Green-Lagrange strain tensor, and $\mathbf{f}_b$ is the vector of body forces. The second term in the preceding equation can be recognized as the virtual work of the elastic forces, it can be rewritten to define the generalized elastic forces, that is

$$\delta W_s = \int_V \boldsymbol{\sigma}_{P2} : \delta\boldsymbol{\varepsilon}\, dV = \mathbf{Q}_s^T \delta\mathbf{e} \tag{18}$$

where $\delta\mathbf{e}$ is the virtual change in the nodal coordinates associated with a particular ANCF finite element or a body, and $\mathbf{Q}_s$ is the vector of the generalized elastic forces. The vector of elastic forces often takes a fairly complicated form, especially in the case of plasticity formulations, and is obtained using numerical integration methods. The principle of virtual work leads to the following equations of motion:

$$\mathbf{M}\ddot{\mathbf{e}} + \mathbf{Q}_s - \mathbf{Q}_e = \mathbf{0} \tag{19}$$

where $\mathbf{M}$ is the symmetric mass matrix, and $\mathbf{Q}_e$ is the vector of body applied nodal forces.

Knowing the strains, the soil properties, yield function, and the flow rule; the state of soil deformation (elastic or plastic) can be determined. Knowing the state of deformation, the constitutive model appropriate for this state can be used to determine the elastic force vector $\mathbf{Q}_s$.

5 COMPARISON OF TWO MODELS

Continuum-based and terramechanics soil models have been used in various vehicle-terrain interaction studies. Through these studies and the overview presented above, one can develop a comparison of the two types of models.

Unlike continuum based models, terramechanics models assume a contact interface appropriate for wheel-terrain or track-terrain interfaces. These assumed contact interfaces can take into consideration contact between a flexible wheel and the terrain through the use of concepts as effective wheel diameter, elliptic soil contact surface, etc. Continuum models do not have this limitation, they allow for modeling contact between arbitrary surfaces while capturing higher modes of deformation.

As is evident from the above discussion, there exists a wide variety of continuum soil models each with its own merits and range of applicability. Models based on the concept of the yield surface offer a rational method for improvement and development of new soil constitutive models. Until recently, terramechanics models' range of applicability required models with characteristic dimensions similar to that of an average vehicle. With certain modifications terramechanics models can be made relevant to the simulation of vehicles and robots of much smaller dimensions.

While there are plenty of differences between terramechanics and continuum-based soil models, there also exist some similarities between them. For example, both models can be fitted to experimental data. Standard techniques have been established for determining the coefficients that appear in both model types. However, some continuum models require extensive testing in order to generate appropriate coefficients to fit experimental results. Also, both categories of models can be modified to capture viscous effects.

While this paper focuses on discussing the basic differences between the terrmechanics and the continuum based soil models, numerical studies that compare the two methods will be presented by the authors in future investigations.

6 SUMMARY

In this paper, several simple models including Bekker's model are discussed. Bekker's model as well as other parametric and analytical terramechanics models have been used in the study of track/soil interaction and can be implemented in MBS algorithms using simple discrete force elements. More general continuum plasticity soil formulations are reviewed. The absolute nodal coordinate formulation (ANCF) which allows for the study of vehicle/soil interaction is also discussed.

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